Math 3 Name ____________________________

Circles Review

Use the relationships between circular revolutions, degrees, and radians to complete the following. Show your work!

1. 72 degrees = $\frac{2\pi}{5}$ Radians = $\frac{1}{5}$ Revolutions

$72\left(\frac{\pi}{180}\right)$ $\frac{72}{360}$

2. $\frac{9\pi}{10}$ radians = 162° Degrees = $\frac{9}{20}$ Revolutions

$\left(\frac{9\pi}{10}\right)\left(\frac{180}{\pi}\right)$ $\frac{162}{360}$

3. 1.6 Revolutions = 576° degrees = $\frac{16\pi}{5}$ Radians

$\frac{\text{Degree}}{360} = \text{rev}$ $(1.6)(360)$ $576\left(\frac{\pi}{180}\right)$

$\text{rev} \cdot 360 = \text{Degrr}$

$180-\theta$ $\pi-\theta$	θ
$\theta-180$ $\theta-\pi$	$360-\theta$ $2\pi-\theta$

Find each reference angle

4. 175°

$180-175$

5°

5. $\frac{41\pi}{12} - \frac{24\pi}{12} = \frac{17\pi}{12}$

$\frac{17\pi}{12} - \pi$

$\frac{17\pi}{12} - \frac{12\pi}{12} = \frac{5\pi}{12}$

6. $-83^\circ + 360$

277°

$360-277$

83°

7. $-\frac{17\pi}{9}$

$-\frac{17\pi}{9} + \frac{18\pi}{9} = \boxed{\frac{\pi}{9}}$

Find the exact value for each

8. $\cos 120^\circ = -\frac{1}{2}$

9. $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

10. $\tan\frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$

$\frac{y}{x} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$

11. $\cos 510^\circ$

$510-360$

150

$\cos 150^\circ$

$-\frac{\sqrt{3}}{2}$

$-\frac{3\pi}{4} + \frac{8\pi}{4} = \frac{5\pi}{4}$ $-135+360$

12. $\sin -\frac{3\pi}{4}$

$\sin\frac{5\pi}{4}$

$-\frac{\sqrt{2}}{2}$

13. $\tan -135^\circ$

$\tan 225^\circ$

$\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$

14. $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

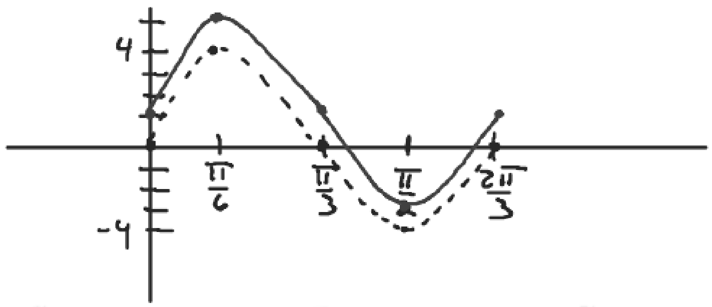
15. $\cos 315^\circ = \frac{\sqrt{2}}{2}$

Provide the given information and then graph each equation. **MAKE SURE YOU ACCURATELY LABLE YOUR X AND Y AXIS.**

16. $y = 1{+}4\sin 3x$

$y = 4\sin 3x + 1$

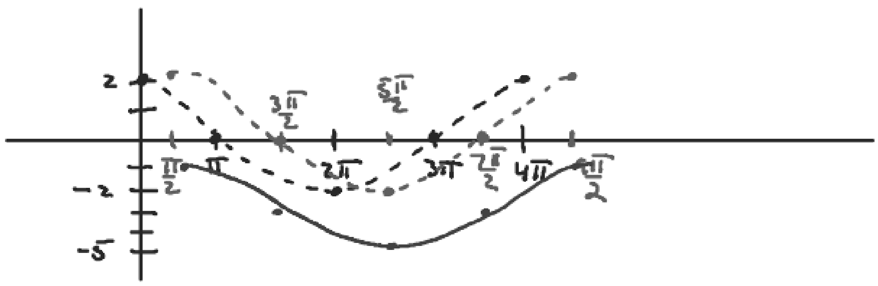
Amplitude 4	Period $2\pi/3$	Phase Shift None
Vertical Shift up 1		



17. $y = 2\cos\frac{1}{2}\left(x - \frac{\pi}{2}\right) - 3$

$Per\ \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

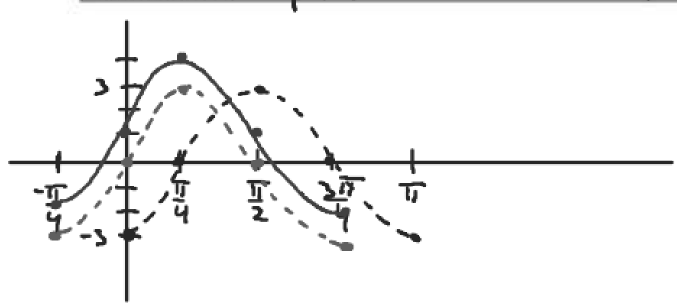
Amplitude 2	Period 4π	Phase Shift $\frac{\pi}{2}$ Right
Vertical Shift Down 3		



18. $y = -3\cos 2(x + \frac{\pi}{4}) + 1$

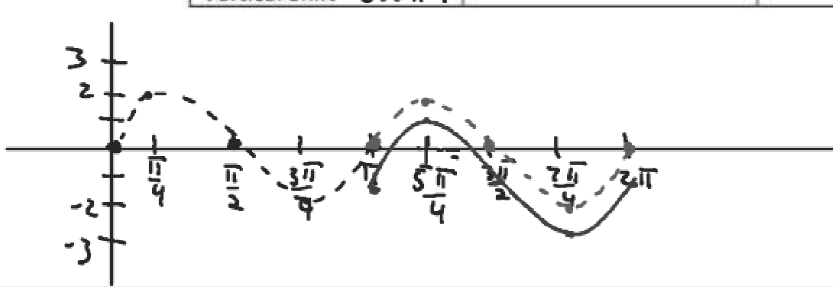
$Per\ \frac{2\pi}{2} = \pi$

Amplitude 3	Period π	Phase Shift $\frac{\pi}{4}$ Left
Vertical Shift Up 1		



19. $y = 2\sin 2(x{-}\pi) - 1$

Amplitude 2	Period π	Phase Shift Right π
Vertical Shift Down 1	1	



$y = A\sin B(x-c)+D$

20. Write the equation of the SINE curve that has a period of 6π, a maximum at 2, a minimum at – 2, and a phase shift of π to the right.

$y = 2\sin\frac{1}{3}(x-\pi)$

$Amp = \frac{max - min}{2}$

$= \frac{2-(-2)}{2} = 2$

Per 6π $\quad B = \frac{2\pi}{Per} \quad \frac{2\pi}{6\pi} = \frac{1}{3}$

21. Write the equation of the COSINE curve that has an amplitude of 4, a period of $\frac{3\pi}{2}$, a phase shift of $\frac{\pi}{4}$ left and a vertical shift down 2.

$y = A\cos B(x-c)+D$

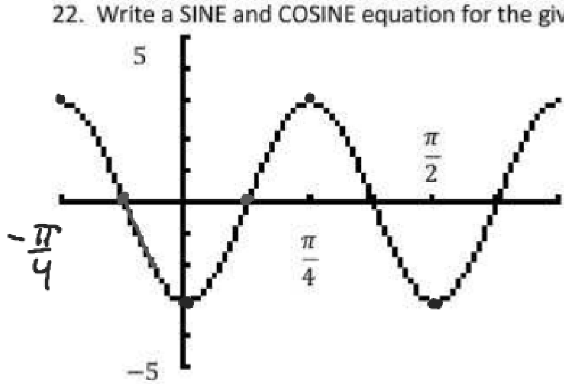
$A = 4$

$B = \frac{2\pi}{Per} = \frac{2\pi}{\frac{3\pi}{2}} = \frac{2\pi}{1}\cdot\frac{2}{3\pi} = \frac{4}{3}$

$y = 4\cos\frac{4}{3}\left(x-\frac{\pi}{4}\right)-2$

22. Write a SINE and COSINE equation for the given graph.

Could be more than one correct answer



Amp = 3

Per $\frac{\pi}{2}$

$B = \frac{2\pi}{\frac{\pi}{2}} = 4$

$y = 3\sin 4\left(x-\frac{\pi}{8}\right)$

$y = -3\sin 4\left(x+\frac{\pi}{8}\right)$

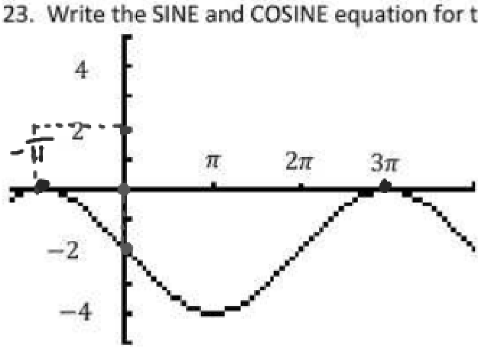
Cosine

$y = 3\cos 4\left(x-\frac{\pi}{4}\right)$

$y = 3\cos 4\left(x+\frac{\pi}{4}\right)$

$y = -3\cos 4x$

23. Write the SINE and COSINE equation for the given graph.



$Amp = \frac{max - min}{2} = \frac{0-(-4)}{2} = \frac{4}{2} = 2$

Per 4π

$B = \frac{2\pi}{4\pi} = \frac{1}{2}$

$y = 2\cos\frac{1}{2}(x+\pi)-2$

V.S. $\frac{max + min}{2} = \frac{0+(-4)}{2} = -2$

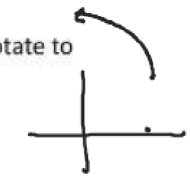
$y = -2\sin\frac{1}{2}x - 2$

24. The Ferris wheel on Navy Pier in Chicago has 60 equally spaced gondolas. Passengers load the Ferris wheel from a platform above the ground. After loading the passengers, the Ferris wheel moves in a counterclockwise direction.

a. There are spokes connecting each gondola to the center of the wheel. What is the measure of the angle formed by adjacent spokes that connect each gondola to the center of the wheel if the angle is measured in degrees? In radians? Show your work.

Angle measure in degrees: 6° $\frac{360}{60}$ *Angle measure in radians:* $\frac{\pi}{30}$

b. Sydney begins in a gondola at the "3 o'clock" position of the Ferris wheel. How far must she rotate to reach the highest position on the Ferris wheel? Give your answer in degrees and radians.



Degrees: 90° *Radians:* $\frac{\pi}{2}$

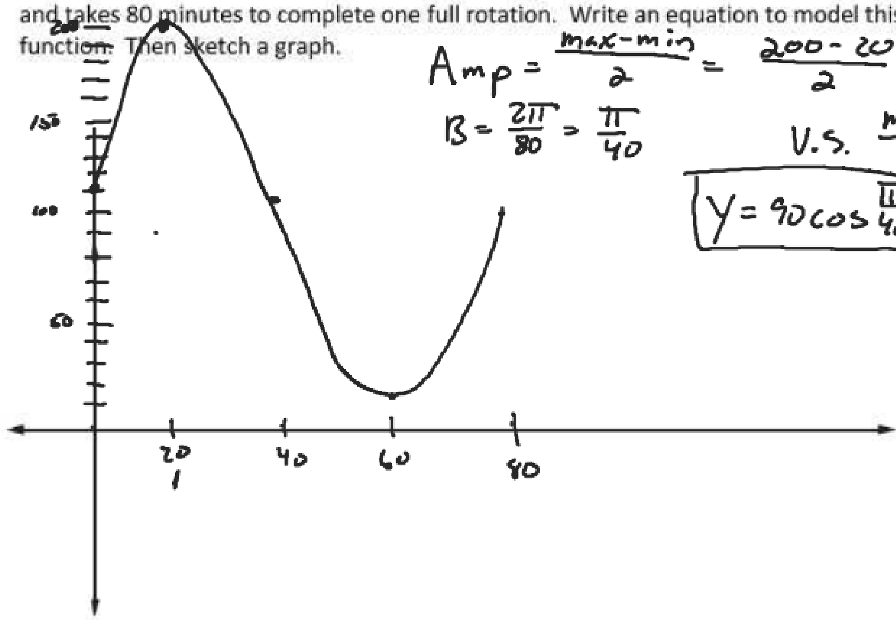
c. If Sydney begins at the "3 o'clock" position and the maximum height she will reach is 200 feet. At the lowest part of the ride she will be 20 feet off the ground. The Ferris wheel moves at a constant rate and takes 80 minutes to complete one full rotation. Write an equation to model this sinusoidal function. Then sketch a graph.

$\text{Amp} = \frac{\max - \min}{2} = \frac{200-20}{2} = 90$

$B = \frac{2\pi}{80} = \frac{\pi}{40}$

$V.S. \; \frac{\max + \min}{2} = \frac{200+20}{2} = 110$

$\boxed{y = 90\cos\frac{\pi}{40}x + 110}$



3:05 pm

d. If Sydney enters the ride at 2:45 pm, what time will she reach the maximum height?

25. Suppose that the height in feet of a Ferris wheel seat changes in a pattern that can be modeled by the function $h(t) = 25 + 7\sin t$, where t is time in minutes since the wheel started turning.

a. What is the radius of the Ferris wheel?

Radius = 7 ft

$y = 25 + 7\sin t$

$y = 7\sin t + 25$

b. Determine the maximum height of a seat on this Ferris wheel. Show your work.

$25 + 7 = 31$ ft

c. If the Ferris wheel is operating without stopping, how long will it take a seat to move from the highest point on the wheel all the way around the circle and back to the highest point?

Per $\frac{2\pi}{B} = \frac{2\pi}{1}$ 6 min 17 sec

$= 6.28$ min $.28(60) \approx 17$

26. An equation in the form $y = A\sin Bx$ has period 4π and Amplitude 8.

a. Find A and B. Explain your reasoning.

A = 8 B = $\frac{1}{2}$

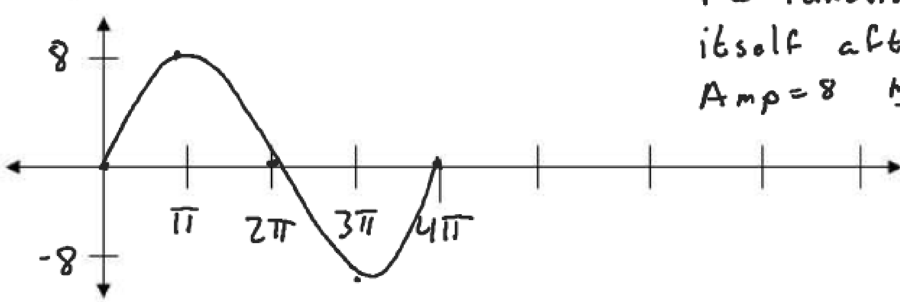
Amp = A

$B = \frac{2\pi}{Per} = \frac{2\pi}{4\pi} = \frac{1}{2}$

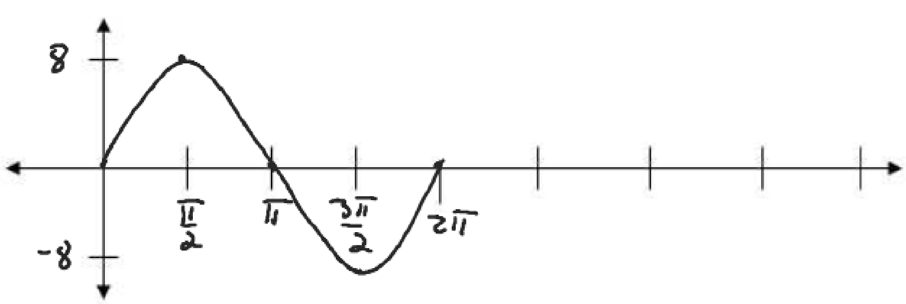
b. Graph the function in Part *a*. Explain how you can see form the graph that the period is 4π and the amplitude is ~~4~~ 8

The function repeats itself after 4π.

Amp = 8 $\frac{max - min}{2} = 8$



c. Change on numbe in the above equation so the period is 2π. Write the new equation and sketch the resulting graph. $B = \frac{2\pi}{2\pi} = 1$ $y = 8\sin x$



$\frac{15+5}{2} = 10$

Per 12.4

$B = \frac{2\pi}{12.4} = \frac{\pi}{6.2}$

V.S. (D)

26. Tides go up and down during a 12.4 hour period (half lunar day). The average depth of a certain river is 10 m and ranges from a low tide of 5 m to a high tide of 15 m. The variation can be approximated by a sinusoidal curve.

$Amp = \frac{15-5}{2} = 5$

a) Write an equation that gives the approximate variation y, if x is the number of hours after midnight if high tide occurs at 9:00 am.

$y = A\cos B(x-c) + D$

$= 5\cos \frac{\pi}{6.2}(x-9) + 10$

b) Determine the height of the tide at 2 pm.

2 pm = 14

14.995 m